FP1 Series Questions

3 Show that

$$\sum_{r=1}^{n} (r^2 - r) = kn(n+1)(n-1)$$

where *k* is a rational number.

(4 marks)

- 6 (a) (i) Expand $(2r-1)^2$. (1 mark)
 - (ii) Hence show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2-1)$$
 (5 marks)

(b) Hence find the sum of the squares of the odd numbers between 100 and 200.

(4 marks)

FP1 Series Answers

3	$\Sigma(r^2 - r) = \Sigma r^2 - \Sigma r$	M1		
	At least one linear factor found	m1		
	$\Sigma(r^2 - r) = \frac{1}{6}n(n+1)(2n+1-3)$	m1		OE
	$\dots = \frac{1}{3}n(n+1)(n-1)$	A1	4	
	Total		4	
6(a	(i) $(2r-1)^2 = 4r^2 - 4r + 1$	B1	1	
	(i) $(2r-1)^2 = 4r^2 - 4r + 1$ (ii) $\sum (2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$ $\dots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	M1		
	$\dots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	m1A1	L	
	$\sum 1 = n$	B1		
	Result convincingly shown	A1	5	AG
	(b) Sum = $f(100) - f(50)$	M1A	1	M1 for 100 ± 1 and 50 ± 1
	=1166 650	A2	4	SC $f(100) - f(51) = 1\ 156\ 449$: 3/4
	Tota	al	10	